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Supergravity Microstates for BPS Black Holes and Black Rings

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Abstract

We demonstrate a solution generating technique, modulo some constraints, for a large class of smooth supergravity solutions with the same asymptotic charges as a five dimensional 3-charge BPS black hole or black ring, dual to a $D1/D5/P$ system. These solutions are characterized by a harmonic function with both positive and negative poles, which induces a geometric transition whereby singular sources have disappeared and all of the net charge at infinity is sourced by fluxes through two-cycles joining the poles of the harmonic function.

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1 Introduction and Summary

The black hole information puzzle [1] is a particularly striking example of the problems encountered when trying to combine quantum mechanics and gravity. Various computations in string theory have suggested that the usual picture of a black hole with an event horizon, then empty space with a central singularity, could be an emergent phenomena arising when we coarse grain over a set of microstates. A similar conjecture applies for the new black ring solutions [2, 3, 4, 5, 6, 7, 8]. Recently, Mathur and Lunin have made a stronger conjecture [9] (see [10] for a recent review) which holds that black hole microstates are characterized by string theory backgrounds with no horizons.

Up to now the main evidence for this conjecture involves finding microstates for two charge proto-black holes with no classical horizon area. Recently, Mathur et al [11, 12, 13] uncovered some three charge supergravity solutions with no horizon (see also [14]). Since these solutions exactly saturate a bound on angular momentum, they have the same asymptotic charges as a black hole whose horizon area classically vanishes. In [15] some finite temperature (non-supersymmetric) generalizations appear for these which correspond to black holes with finite size horizons; we are interested in finding microstates for BPS black holes which have already have finite size horizons for zero temperature.

Bena and Warner [16] (see also [17]) have developed a formalism for finding five dimensional supersymmetric solutions with three asymptotic charges and two angular momenta as reductions from M-theory. We will show how a general class of these solutions can be layed out which corresponds (modulo some consistency conditions) to $U(1)$ invariant microstates with the asymptotics of black objects with finite areas, dual to the $D1/D5/P$ system. We develop the simplest cases; some too simple, in fact, to look like black objects with non-vanishing horizons. The framework we lay out, however, exhibits strong potential for finding $U(1)$-invariant supergravity microstates for all five-dimensional black objects.

The Bena-Warner ansatz involves a fibration of the time coordinate over a hyperkahler base space. The key insight from [13] is that even if this base space is singular, a time fibration can be formulated such that the total space is completely smooth. This gives us access to a whole new class of base spaces: new two-cycles replace the origin of $\mathbf{R}^4$.

We will review the Bena-Warner ansatz in section 2 and then in section 3 we will offer a general form for a solution to these equations (a similar set of ansatz appears in [18, 19, 20, 21, 6, 22]) which are obviously smooth everywhere except at some orbifold points and possibly where the base space is singular. In section 4 we will demonstrate that our solution is smooth everywhere with no closed timelike curves or event horizons given three general sets of constraints. In section 5 we outline a simple class of examples for our formalism related

to the solutions in [11, 12, 13]. They saturate a bound on rotation, and so correspond to microstates of black holes with no classical horizon. In section 6 we discuss how these examples can be generalized so as to include microstates for black holes which have finite size horizons. Finally we will briefly discuss further generalizations and future directions.

Work concurrent with this paper is also appearing in [23].

2 The Bena-Warner Ansatz and System of Equations

In [16] Bena and Warner neatly lay out an ansatz for a 1/8 BPS solution with three charges in five dimensions as a reduction from M-theory where the charges come from wrapping membranes on three separate T^2's used to reduce from eleven dimensions. The five dimensional space is written as time fibered over a hyperkahler base space, HK. The resulting M-theory metric takes the form:

$$ds_{11}^2 = -(Z_1 Z_2 Z_3)^{-2/3}(dt+k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds_{HK}^2 + ds_{T^6}^2, \tag{2.1}$$

where

$$ds_{T^6}^2 = (Z_1 Z_2 Z_3)^{1/3}\left(Z_1^{-1}(dz_1^2+dz_2^2) + Z_2^{-1}(dz_3^2+dz_4^2) + Z_3^{-1}(dz_5^2+dz_6^2)\right). \tag{2.2}$$

The Z_i's and k are functions and one-form respectively on the hyperkahler base space, the three T^2's have volumes V_i. The gauge field takes the form:

$$\begin{aligned} C_{(3)} &= -(dt+k)\left(Z_1^{-1}\, dz_1\wedge dz_2 + Z_2^{-1}\, dz_3\wedge dz_4 + Z_3^{-1}\, dz_5\wedge dz_6\right) \\ &\quad +2\, a_1\wedge dz_1\wedge dz_2 + 2\, a_2\wedge dz_3\wedge dz_4 + 2\, a_3\wedge dz_5\wedge dz_6, \end{aligned} \tag{2.3}$$

where the a_i are one-forms on the base space. After reduction on T^6 the C-field effectively defines three separate $U(1)$ bundles, with connections $\mathcal{A}_i = -(dt+k)Z_i^{-1} + 2\, a_i$, on the 5-dimensional total space. If we now define the two-forms,

$$G_i = da_i, \tag{2.4}$$

[16] show that the equations of motion reduce to the three conditions (here the Hodge operator refers only to the base space HK):

$$G_i = \star G_i, \tag{2.5}$$

$$d\star dZ_i = 2s^{ijk}\, G_j\wedge G_k, \tag{2.6}$$

$$dk + \star dk = 2\, G_i Z_i. \tag{2.7}$$

where we define the symmetric tensor $s^{ijk} = |\epsilon^{ijk}|$.

3 Solving the Equations and Asymptotics

In this section, we will first solve the Bena-Warner ansatz by selecting a base space HK which has the property that the reduced five dimensional space is asymptotically flat. Written in Gibbons-Hawking form, the metric for this base space is:

$$ds^2_{HK} = H^{-1}\sigma^2 + H(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2), \tag{3.1}$$

where H is a positive function on $\mathbf{R}^3$ with integer poles and σ is a one form on $\mathbf{R}^3$ of the form $d\tau + f_a dx^a$ (τ has period 4π) satisfying

$$\star_3 \, d\sigma = dH. \tag{3.2}$$

Asymptotic flatness with no ADE identification basically forces the choice $H = 1/r$ on us, this is fairly limiting. Following [13], however, we know that the time fibration can allow us to relax the hyperkahler condition on HK. We allow a singular pseudo-hyperkahler HK (see [13] for a definition) so long as the total space with the time-fiber is smooth.

Our complete solution will be encoded using a set of 8 harmonic function on $\mathbf{R}^3$: H, h_i, M_i and K, generalizing the appendix in [13] patterned on [24]. These can take a fairly generic form, with constraints and relations that we will gradually lay out and then summarize.

3.1 The function H

Relaxing the hyperkahler conditions provides us with more general candidates for H, such as:

$$H = \sum_{p=1}^{N} \frac{n_p}{r_p}, \qquad r_p = |\vec{r}_p| = |\vec{x} - \vec{x}_p|, \qquad \sum_{p=1}^{N} n_p = 1. \tag{3.3}$$

The last sum guarantees that asymptotically we have an $\mathbf{R}^4$. Relaxing the condition that the n_p be positive has allowed us to introduce an arbitrary number of poles, N, at positions $\vec{x}_p$. We can use this potential to divide $\mathbf{R}_3$ into a part where $H > 0$ (Region I) and a compact (but not necessarily connected) part where $H < 0$ (Region II). The two are separated by a domain wall where $H = 0$ (from now we will refer to this as "the domain wall"). The metric in eq.(3.1) is singular at this domain wall, but we will see in the next section how the t-fibration saves the day. It does not, however, change the fact that with our choice of H there are two-cycles, S^{pq}, coming from the fiber σ over each interval from $\vec{x_p}$ to $\vec{x_q}$; it just makes them non-singular.

For convenience we define the following quantities:

$$\Pi^r = \prod_{p=1}^{N} r_p, \quad \Pi^r_p = \prod_{q \neq p} r_q, \quad \Pi^r_{ps} = \prod_{s \neq q \neq p} r_q, \qquad f = \sum_{p=1}^{N} n_p \Pi^r_p. \tag{3.4}$$

Hence, H can be written as f/Π^r. Note also that eq.(3.3) implies that $f \to r^{N-1}$ at asymptotic infinity, regardless of our choice of origin.

3.2 The Dipole Fields

To generate the G_i, it is useful to define harmonic functions on HK that, up to a gauge transformation, fall off faster than $1/r$ as $r \to \infty$. To limit the scope of our discussion, we choose these to have poles in the same place as H[1].

$$h_i = \sum_{p=1}^{N} \frac{d_i^{(p)}}{4r_p}, \quad \sum_{p=1}^{N} d_i^{(p)} = 0. \tag{3.5}$$

With these functions we define self-dual G_i's:

$$G_i = d(h_i/H) \wedge \sigma - H \star_3 d(h_i/H), \tag{3.6}$$

which have the requisite fall-off ($\star_3$ is the Hodge operator on $\mathbf{R}^3$). Note that we can always satisfy the condition in eq.(3.5) by applying the gauge transformation

$$h_i \to h_i - \frac{1}{4} \sum_p d_i^{(p)} H. \tag{3.7}$$

We can integrate the two-forms partially to get an expression for the a_i's:

$$a_i = (h_i/H)\,\sigma + a_{ia}dx^a, \qquad d(a_{ia}dx^a) = -\star_3 dh_i \tag{3.8}$$

where the dx^a denote any complete set of one-forms on flat $\mathbf{R}^3$. Since we have chosen to localize the poles of the h_i on top of the poles of H, these one-forms have no singularities except at $H = 0$. It useful to define the dipole moments:

$$\vec{D}_i = \sum_{p=1}^{N} d_i^{(p)} \vec{x}_p, \tag{3.9}$$

which control the asymptotics of our solution. Note that a priori these need not be parallel vectors such as in the asymptotics of a single black ring. Also, since $\sum_p d_i^{(p)} = 0$, these dipole moments are independent of the choice of origin for $\mathbf{R}^3$. It is also useful to define, more locally, the relative dipole moments and to rewrite expressions as a function of these quantities:

$$d_i^{pq} = n_p d_i^{(q)} - n_q d_i^{(p)} \quad \Rightarrow \qquad d_i^{(p)} = -\sum_{q=1}^{N} d_i^{pq}, \qquad \vec{D}_i = -\frac{1}{2} \sum_{pq} d_i^{pq}\,(\vec{x}_p - \vec{x}_q). \tag{3.10}$$

The d_i^{pq}'s will appear almost everywhere in our solution. As we will see, they measure the various $U(1)$ fluxes through the two-cycle, S^{pq}, connecting $\vec{x}_p$ and $\vec{x}_q$.

[1] It is our sense that poles in other places lead to superpositions with previously known solutions such as supertubes and AdS throats and so would distract attention from the new solutions we present

3.3 The Monopole fields

The membrane charge at infinity can be read off from the C-field components with a time component. Looking at eq.(2.3) this means that the Z_i's encode the three membrane charges Q_i, therefore they must have a falloff like

$$Z_i \to 1 + \frac{Q_i}{4r} \qquad \text{as} \qquad r \to \infty. \tag{3.11}$$

since the asymptotic $\mathbf{R}^4$ radial coordinate is $R = 2r^{\frac{1}{2}}$. Taking advantage of the natural radial distances in H we can write arbitrary new harmonic functions,

$$M_i = 1 + \sum_{p=1}^{N} \frac{Q_i^{(p)}}{4r_p}, \qquad \sum_{p=1}^{N} Q_i^{(p)} = Q_i, \tag{3.12}$$

which have exactly the asymptotics above. We use these to write down the following ansatz for the Z_i's

$$Z_i \;\; = \;\; M_i + 2\, H^{-1} s^{ijk} h_j\, h_k. \tag{3.13}$$

The correction terms fall off like $1/r^2$ so they don't mess up the asymptotics. They are necessary to satisfy the second equation of motion, eq.(2.6):

$$d{\star}dZ_1 = 4\, G_2 \wedge G_3, \qquad d{\star}dZ_2 = 4\, G_1 \wedge G_3, \qquad d{\star}dZ_3 = 4\, G_1 \wedge G_2, \tag{3.14}$$

up to some extra delta function sources. For the purposes of this paper, we will focus our attention on Z_i's tuned so that they have no singularities apart from those at $H = 0$. This removes the delta function sources in eq.(3.14) and allows us to write the $Q_i^{(p)}$ explicitly as:

$$Q_i^{(p)} = -\frac{s^{ijk}}{2n_p}\, d_j^{(p)}\, d_k^{(p)}. \tag{3.15}$$

We can now rewrite the Z_i in the following form:

$$Z_i = 1 - \frac{s^{ijk}}{4f} \sum_{p,q} \frac{d_j^{pq} d_k^{pq} \Pi^r_{pq}}{4n_p n_q}, \tag{3.16}$$

where we have used the quantities f and Π^r_p from eq.(3.4). This implies an alternate form for the Q_i,

$$Q_i = \sum_{p,q} Q_i^{pq}, \qquad \text{where} \qquad Q_i^{pq} = -\frac{s^{ijk} d_j^{pq} d_k^{pq}}{4n_p n_q}, \tag{3.17}$$

which makes clear that we can also think of the total charge at infinity as coming from a sum of contributions from each two-cycle S^{pq}.

3.4 The Angular Momentum

Finally, lets look at the angular momentum. We use the natural basis

$$k = k_0\, \sigma + k_a\, dx^a, \tag{3.18}$$

with the $d\alpha^i$ a complete basis of one-forms on $\mathbf{R}^3$. Our ansatz is now:

$$\begin{aligned} k_0 &= K + 8H^{-2}\, h_1\, h_2\, h_3 + H^{-1}\, M_1\, h_1 + H^{-1}\, M_2\, h_2 + H^{-1}\, M_3\, h_3 \qquad (3.19)\\ &= K - 4H^{-2}\, h_1\, h_2\, h_3 + Z_i\,(h_i/H) = \bar{K} + Z_i\,(h_i/H) \\ \star_3 d(k_a dx^a) &= H dK - K\, dH + h_i\, dM_i - M_i\, dh_i \qquad (3.20)\\ &= H\, d\bar{K} - \bar{K}\, dH + h_i\, dZ_i - Z_i\, dh_i \end{aligned}$$

Here we have defined a new harmonic function, K, and its partner function, $\bar{K}$:[2]

$$K = \sum_{p=1}^{N} \left(\frac{\ell_p}{r_p}\right), \qquad \bar{K} = K - 4H^{-2}\, h_1\, h_2\, h_3. \tag{3.21}$$

The regularity of k is important here. There is an integrability condition on eq. (3.20) which basically requires that "$d\star_3$" of that equation is zero; this is trivially satisfied everywhere except at the poles of our harmonic functions. We would like the one-form k to have no singularities except at $H = 0$ and for $k_a dx^a$ to be a globally well defined one-form everywhere on $\mathbf{R}^3$ with an asymptotic fall-off like $1/r$. This turns out to be possible if we use the freedom to add any closed form of our choice to $k_a dx^a$, and demand:

$$\ell_p = \frac{d_1^{(p)} d_2^{(p)} d_3^{(p)}}{16 n_p^2}, \qquad k_0|_{r_p=0} = 0. \tag{3.22}$$

The first condition removes poles in $\bar{K}$, the second condition insures that $d^2(k_a dx^a = 0)$ everywhere, i.e. that $k_a dx^a$ is globally well defined. With the first condition we can rationalize our form for k_0 a little bit:

$$\begin{aligned} k_0 \;=\; & \sum_{p=1}^{N} \sum_i \frac{d_i^{(p)} \Pi_p^r}{4f} + \frac{1}{16 \Pi^r f^2} \Bigg[\sum_{p,q,s} \left(d_1^{(p)} d_2^{(p)} d_3^{(p)} \frac{n_q n_s}{n_p^2} - d_1^{(p)} d_2^{(q)} d_3^{(s)} \right) \Pi_p^r \Pi_q^r \Pi_s^r \qquad (3.23)\\ & - s^{ijk} \sum_{p=1}^{N} d_i^{(p)} \Pi^r \Pi_p^r \sum_{q,s} \frac{d_j^{qs} d_k^{qs} \Pi_{qs}^r}{4 n_q n_s} \Bigg] \end{aligned}$$

This allows us to solve for $k_0 = 0$ at the points $r_p = 0$, boiling down to

$$0 = \sum_i d_i^{(p)} \; + \sum_q \frac{1}{4 n_p^2 n_q^2} \frac{1}{r_{pq}} \prod_i d_i^{pq} \tag{3.24}$$

[2] ℓ_p should not be confused with the Planck length ℓ_P.

where $r_{pq} = |\vec{x}_p - \vec{x}_q|$. This puts at most $N-1$ independent constraints on the relative positions of the poles. Of course, all the r_{pq} have to be non-negative, therefore a bad choice of d_i^{pq} may lead to no solution at all! Note, also, that if any of the d_i^{pq} vanishes, then the corresponding r_{pq} will not appear in these constraints.

Using eqs.(3.22) and (3.24), we can rewrite eq.(3.20) as:

$$\star_3\, d(k_a dx^a) = -\sum_{p,q} \frac{1}{32 n_p^2 n_q^2} \prod_i \frac{d_i^{pq}}{r_{pq} r_q^2 r_p^2} \Big((r_{pq} - r_p) r_p dr_q - (r_{pq} - r_q) r_q dr_p \Big) \tag{3.25}$$

If we define ϕ_{pq} as the right-handed angle about the directed line from $\vec{x_p}$ to $\vec{x_q}$, we can integrate the expression above explicitly to get:

$$k_a dx^a = \sum_{p,q} \frac{1}{64 n_p^2 n_q^2} \prod_i \frac{d_i^{pq}}{r_{pq}^2 r_q r_p} (r_p + r_q - r_{pq}) \Big(r_{pq}^2 - (r_p - r_q)^2 \Big)\, d\phi_{pq}. \tag{3.26}$$

The two-form dk naturally splits into a self-dual and anti-self-dual part. The split gives:

$$dk_L = (dk + \star\, dk)/2 \;=\; Z_i G_i \tag{3.27}$$

$$dk_R = (dk - \star\, dk)/2 \;=\; (d\bar{K} \wedge \sigma + H \star_3 d\bar{K}) + (h_i/H)\,(dZ_i \wedge \sigma + H \star_3 dZ_i) \tag{3.28}$$

Notice that we have correctly solved eq.(2.7) and also that K contributes only to the anti-self-dual part of dk. Looking at the asymptotics, these tensors look like:

$$\begin{aligned} dk_L \;&\to\; \sum_i G_i \to \frac{1}{4}\left[d\left(\frac{\sum_i 4\vec{D}_i \cdot \hat{r}}{4r} \right) \wedge \sigma - \frac{1}{4r} \star_3 d \left(\frac{\sum_i 4\vec{D}_i \cdot \hat{r}}{r} \right) \right] \\ dk_R \;&\to\; \sum_{p=1}^{N} 16 \ell_p \cdot \frac{1}{4}\Big[d\left(\frac{1}{4r}\right) \wedge \sigma + \frac{d\sigma}{4r} \Big] \end{aligned} \tag{3.29}$$

From these expressions we can read the magnitudes of the angular momenta as measured at infinity. They take the values :

$$J_L \;=\; \frac{4G_5}{\pi} j_L = |4 \sum_i \vec{D}_i|, \tag{3.30}$$

$$J_R \;=\; \frac{4G_5}{\pi} j_R = 16 \sum_{p=1}^{N} \ell_p, \tag{3.31}$$

The $j_{L,R} \in \mathbf{Z}$ and G_5 is the five-dimensional Newton's constant, which has length dimensions three and takes the value $G_5 = 16\pi^7 \ell_P^9 \prod_i V_i^{-1}$.

4 Smoothness of the Solution

So far, we have proposed general forms for the G_i, Z_i and k which satisfy the equations (2.5-2.7). The next step is to demonstrate that we can exhibit solutions without any singularities.

Since our base space HK is singular when $H \to 0$ it seems natural to check that the full metric and the C-field are free of singularities near this domain wall. In this section, we will demonstrate how, with the most general harmonic functions, we avoid any singularities at the domain wall. We will also address other potential pitfalls which might render our solution unphysical.

Assuming that none of the poles in the h_i, M_i and K overlap with the domain wall, we see that a_i, Z_i and k have the following expansions as $H \to 0$:

$$\begin{aligned} a_i &= (h_i|_0 H^{-1})\,\sigma + \mathcal{O}(H^0), \\ Z_i &= Z_{i,-1}H^{-1} + \mathcal{O}(H^0) = 2s^{ijk}h_j|_0 h_k|_0\, H^{-1} + \mathcal{O}(H^0), \\ \sigma &= \sigma_0 + H\sigma_1 + \mathcal{O}(H^2), \\ k &= (k_{0,-2}H^{-2})\,\sigma + (k_{0,-1}\sigma)H^{-1} + k_{0,0}\,\sigma + k_a|_0 dx^a + \mathcal{O}(H). \end{aligned} \tag{4.1}$$

For notational simplicity in section 4.1 we will refer to all non-singular variables (such as k_a or h_i) by there values at $H = 0$ without including a subscript.

4.1 The C-field and Metric as $H \to 0$

A quick inspection of the C-field in eq.(2.3) near the domain wall shows that potential singularities appear only in the purely spatial part, and only at order H^{-1}. The dangerous terms are of the form

$$H^{-1}\left(2h_i - (Z_i^{-1}k_{0,-2}\right)\sigma_0 \wedge \mathrm{Vol}_{T_i^2} \tag{4.2}$$

This singular term goes like (with no sum on i):

$$2h_iH^{-1} - s^{ijk}H\,(2h_jh_k + \mathcal{O}(H))^{-1}\,H^{-2}\left(4s^{ijk}h_ih_jh_k + \mathcal{O}(H)\right) = 0 \cdot H^{-1} + \mathcal{O}(H), \tag{4.3}$$

so it cancels out generically. The metric has singularities of leading order H^{-2} and subleading order H^{-1}, coming from the part of the metric which contains σ

$$-\,(Z_1Z_2Z_3)^{-2/3}\,k^2 + (Z_1Z_2Z_3)^{1/3}H^{-1}\sigma^2 = (Z_1Z_2Z_3)^{-2/3}\left(-k^2 + (Z_1Z_2Z_3)H^{-1}\sigma^2\right) \tag{4.4}$$

Up to a finite coefficient, the singular piece is proportional to:

$$H^{-2}\left(-k_{0,-2}^2 + (Z_1Z_2Z_3)|_{-3}\right)(\sigma_0^2 + 2H\sigma_1\sigma_0) + H^{-1}\left(-2k_{0,-2}k_{0,-1} + (Z_1Z_2Z_3)|_{-2}\right)\sigma_0^2 \tag{4.5}$$

We can see that these singular terms vanish for our ansatz, since:

$$\begin{aligned} -\,(k_{0,-2})^2 + (Z_1Z_2Z_3)|_{-3} &= -64h_1^2h_2^2h_3^2 + (4h_2h_3)(4h_1h_3)(4h_1h_2) = 0, \\ -2k_{0,-2}k_{0,-1} + (Z_1Z_2Z_3)|_{-2} &= -16h_1h_2h_3(M_ih_i) + (M_1(4h_1h_3)(4h_1h_2) + \text{perms}) = 0. \end{aligned} \tag{4.6}$$

4.2 Zeroes of the Z_i

Looking at eq.(2.3) or the determinant of the metric, $\sqrt{-g_{11}} = (Z_1 Z_2 Z_3)^{1/3}\, H\, \sqrt{g_{\mathbf{R}^3}}$, we see that to avoid singularities it is necessary for $Z_i \neq 0$. We have a simple tactic for enforcing the non-vanishing of the Z_i: we demand that they obey

$$Z_i H > 0 \qquad \forall i \in 1,2,3 \tag{4.7}$$

everywhere. We choose a convention where the Z_i's are negative at infinity, so this means that in Region I the Z_i must remain positive, while in Region II this means that the Z_i must remain negative. This uniformity also implies that $(Z_1 Z_2 Z_3)^{1/3}$ is negative inside and positive outside, and guarantees that combinations like $(Z_1 Z_2 Z_3)^{-2/3}$ and $Z_i^{-1}(Z_1 Z_2 Z_3)^{1/3}$ remain positive everywhere. Using eq. (3.16), we see that the bound above can be written as:

$$4f - s^{ijk} \sum_{p,q} \frac{d_j^{pq} d_k^{pq} \Pi_{pq}^r}{4 n_p n_q} > 0 \qquad \forall i \in 1,2,3. \tag{4.8}$$

In section 5, we will show that with two poles, this bound is automatically satisfied modulo some relative sign conditions on the d_i^{pq}. However, in general eq.(4.8) will provide a non-trivial constraint on the relative positions of the $\vec{x_p}$; if solutions exist these conditions will define boundaries for the moduli space of solutions.

4.3 Closed timelike curves and horizons

Naively the σ fibration has the potential to become timelike, thus creating closed timelike curves. To avoid this, we need to make sure the negative term proportional to k^2 doesn't overwhelm the positive term from the base space. This means that we need to keep:

$$-\,(Z_1 Z_2 Z_3)^{-2/3}\Big(k_0^2 - Z_1 Z_2 Z_3 H^{-1}\Big)\Big) \geq 0, \tag{4.9}$$

so that the norm of the vector in the σ direction remains spacelike. Since the Z_i have been tuned to avoid any poles except at $H = 0$, and since the prefactor above is always positive, we only have to worry about the second term in the product. In general this is a complicated function, however, we can still exclude CTC's along the σ fiber in the neighborhood of the poles of H, $r_p = 0$. There it is easy to see using eq.(3.24) that k_0^2 vanishes faster than r_p, insuring that loops along the σ fiber will remain space-like.

We will exclude more general CTC's in our five dimensional reduced space by requiring that this space be *stably causal*, i.e we will demand that there exists a smooth time function whose gradient is everywhere timelike [25]. Our candidate function is the coordinate t, and it qualifies as a time function if

$$-\,g^{\mu\nu}\partial_\mu t \partial_\nu t = -g^{tt} = (Z_1 Z_2 Z_3)^{-1/3}\, H^{-1}\,\Big((Z_1 Z_2 Z_3)H - H^2\, k_0^2 - g^{ab}_{\mathbf{R}^3} k_a k_b\Big) > 0 \tag{4.10}$$

We will leave for future work the question of possible extra constraints on the relative pole positions which come from this condition.

Granted the time function t, we can now proceed to show that there are no event horizons. The vector ∂_r has a norm,

$$g_{rr} = (Z_1 Z_2 Z_3)^{-2/3} \left((Z_1 Z_2 Z_3) H - k_r^2 \right) \geq -g^{tt}, \tag{4.11}$$

which is positive everywhere due to eq.(4.10). Consider the following vector field:

$$\xi = \left(\frac{g_{rr}}{-g^{tt}} \right)^{1/2} g^{t\mu} \partial_\mu + \epsilon\, \partial_r. \tag{4.12}$$

The norm of ξ is

$$\|\xi\| = -g_{rr} \left(1 - \epsilon^2 \right) \tag{4.13}$$

For $\epsilon < 1$ this is always negative, therefore trajectories generated by this vector field will always be timelike. If we also choose $\epsilon > 0$, these trajectories will always eventually reach asymptotic infinity and so there can be no event horizon.

4.4 Topology of the σ-fibration

The σ-fibration is preserved after the corrections in the full metric. Notice that the base metric has orbifold points with identification on the σ fiber. The order of these points can be determined by the following calculation. For any given two-sphere on the base $\mathbf{R}^3$, we can determine the first Chern class, c_1, of the σ fibration $U(1)$ bundle by integrating $d\sigma$ over that 2-sphere and then using Stoke's theorem to turn that into an integral over the inside B^3:

$$\int_{S^2} d\sigma \;=\; \int_{S^2} \star_3 dH = \int_{B^3} d \star_3 dH = \sum_p \int_{B^3} n_p\, \delta^3(\vec{r} - \vec{r_p}). \tag{4.14}$$

This yields an integer which counts the poles inside of S^2. If this integer is zero, the topology of the σ-fiber over this S^2 is $S^2 \times S^1$, if the integer is ± 1 then the topology is that of S^3. Any larger integer m will give the topology S^3/Z_m.

If we want to understand the corrections to the fibration from the whole metric, we rewrite the σ-fiber piece as:

$$(Z_1 Z_2 Z_3)^{-2/3} (Z_1 Z_2 Z_3 H^{-1} - k_0^2)(\sigma - A k_a dx^a)^2, \qquad A = \frac{H k_0}{Z_1 Z_2 Z_3 - H k_0^2}. \tag{4.15}$$

For a given S^2, the correction to c_1 of the σ bundle is

$$\begin{aligned} \int_{S^2} d(A k_a dx^a) \;&=\; \int_{S^2} (dA \wedge k_a dx^a + A \wedge d(k_a dx^a)) \qquad (4.16) \\ &=\; \int_{B^3} (d^2 A) \wedge k_a dx^a + A \wedge d \star_3 \left(H\, d\bar{K} - \bar{K}\, dH + h_i\, dZ_i - Z_i\, dh_i \right) = 0. \end{aligned}$$

The first of these terms is trivially zero, while the second vanishes due to eq. (3.22). Therefore, there are no corrections to the σ-fiber's topology.

4.5 Topology of the Gauge Fields

Another interesting topological aspect for our solution is the topology of the C-field. We can gain a clearer picture of this by considering a membrane, labelled $\mathcal{M}_i$ wrapped on the torus T_i. This effectively yields a charged particle in the five-dimensional reduced space with charge,

$$e_i = V_i\,\tau_2 = \frac{V_i}{(2\pi)^2 \ell_P^3}, \tag{4.17}$$

which experiences a gauge-field and field strength:

$$\mathcal{A}_i = 2a_i - Z_i^{-1}(dt+k), \qquad F_i = d\mathcal{A}_i. \tag{4.18}$$

For quantum consistency of the wave function for probe charges e_i, we usually require that on the five-dimensional space that the field strength be an integral cohomology class; the properly normalized integral of this class on a regular two-cycle should yield an integer. The compact two-cycles in our geometry, S^{pq}, are represented by line segments on $\mathbf{R}^3$ between two points $\vec{x}_p$ and $\vec{x}_q$ where the function H blows up, along with the fiber σ. Of course, if either n_p or n_q is larger than one, the corresponding S^2 will have orbifold singularities. This means we should look for an integral cohomology class on the universal cover of S^{pq}, i.e. $n_p \cdot n_q$ times our original cycle.

The arguments above lead us to define an integer for each two-cycle, S^{pq}, derived from the following flux integral (all forms are pulled back to the two-cycle):

$$m_i^{(pq)} = n_p\, n_q\, \frac{e_i}{2\pi} \int_{\vec{x}_p}^{\vec{x}_q} \int_\sigma F_i d\tau ds = 2n_p\, n_q\, e_i\, \mathcal{A}_i\Big|_p^q = 2n_p\, n_q\, e_i\,(2a_i - Z_i^{-1}k_0)\Big|_p^q \tag{4.19}$$

To evaluate this, we can use the fact that $k_0 \to 0$ at the points where the σ fiber degenerates. Thus,

$$m_i^{(pq)} = n_p\, n_q\, e_i \left(\frac{d_i}{n_p}\right)\Big|_p^q = e_i\Big(n_p\, d_i^{(q)} - n_q\, d_i^{(p)}\Big) = e_i d_i^{pq}. \tag{4.20}$$

Our story, however, does not end here. Near each orbifold point p, as mentioned above, the local geometry is a cone over $S^3/\mathbf{Z}_{n_p}$. This has $\pi_1 = Z_{n_p}$ and implies that we have the possibility of a discrete Wilson line for each gauge field with a phase of the form $2\pi\, m_i^{(p)}/n_p$ where $m_i^{(p)} \in \mathbf{Z}$ (essentially we are using the fact that $H^2(S^3/\mathbf{Z}_{n_p}) = Z_{n_p}$). The invariance of the local gauge field under a shift of $m_i^{(p)}$ by n_p can be implemented by a gauge transformation similar to that of eq.(3.6). Looking at the gauge field $\mathcal{A}_i$ near one of the orbifold points, for example along the σ fiber, we see that the Wilson line phase is:

$$2\pi\ (4e_i h_i/H)\Big|_p = 2\pi\, m_i^{(p)}/n_p. \tag{4.21}$$

This implies a quantization for the $d_i^{(p)}$ of the form:

$$d_i^{(p)} = m_i^{(p)}/e_i = \frac{(2\pi)^2\ell_P^3}{V_i}\, m_i^{(p)}, \tag{4.22}$$

with several consequences. First, the cohomology requirement in eq.(4.20) is trivially satisfied: $m_i^{(pp')} = n_p m_i^{(p')} - n_{p'} m_i^{(p)}$. Second, unless all the $m_i^{(p)}$ at a given point p are multiples of n_p or on one of it's divisors, the singularity is "frozen" or partially "frozen" [3]. Finally, if we use our formula (3.15) for the monopole charges, we get:

$$Q_i^{(p)} = -\frac{4G_5 e_i}{\pi}\, s^{ijk}\frac{m_j^{(p)}m_k^{(p)}}{2n_p}. \tag{4.23}$$

and so the quantized membrane charge at infinity will be

$$N_i = \frac{\pi}{4e_iG_5}Q_i = \frac{\pi}{4e_iG_5}\sum_{p=1}^{N}Q_i^{(p)} = -\sum_{p=1}^{N}s^{ijk}\frac{m_j^{(p)}m_k^{(p)}}{2n_p} = -\sum_{p,q}s^{ijk}\frac{m_j^{pq}m_k^{pq}}{4n_pn_q}. \tag{4.24}$$

Note that the fact that the N_i's are integers written as a sum of rational numbers add further constraints on the $m_i^{(p)}$'s or m_i^{pq}'s.

4.6 Summary of Conditions

We finish this section by summarizing the exact conditions which will define a smooth (modulo orbifold points) and regular 11-dimensional supergravity solution with three membrane charges and 4 supersymmetries, with no CTC's or event horizons. The solution is completely parameterized by a set of poles on $\mathbf{R}^3$ with quantized residues n_p and quantized fluxes m_i^{pq}. These, and the quantities that depend on them, must satisfy the following conditions:

$$1)\quad \sum_i d_i^{(p)} + \sum_q \frac{1}{4n_p^2n_q^2}\frac{1}{r_{pq}}\prod_i d_i^{pq} = 0, \tag{4.25}$$

$$2)\quad Z_iH > 0 \qquad \forall i \in 1,2,3, \tag{4.26}$$

$$3)\quad (Z_1Z_2Z_3)H - H^2\,k_0^2 - g^{ab}_{\mathbf{R}^3}k_ak_b > 0. \tag{4.27}$$

These need not be independent conditions, for example it may be possible that condition (1) implies (3) implies (2).

There exists a *canonical solution* for eq.(4.25), as long as the d_i^{pq} are such that the r_{pq} come out non-negative, of the form:

$$r_{pq} = \frac{1}{4n_p^2n_q^2}\frac{\prod_i d_i^{pq}}{\sum_i d_i^{pq}}. \tag{4.28}$$

[3] One can see this by reducing the M-theory solution down to IIA on the appropriate circle, then the Wilson line gives mass to otherwise twisted closed strings, or by dualizing to IIB via the relevant T^i's and then the dual circle will be non-trivially fibered so that there is no longer an orbifold point. For similar ideas see [26, 27]

It is not yet clear what extra conditions on the d_i^{pq}, if any, are required for this canonical solution to satisfy the other two constraint equations.

5 The Basic 2-Pole Example

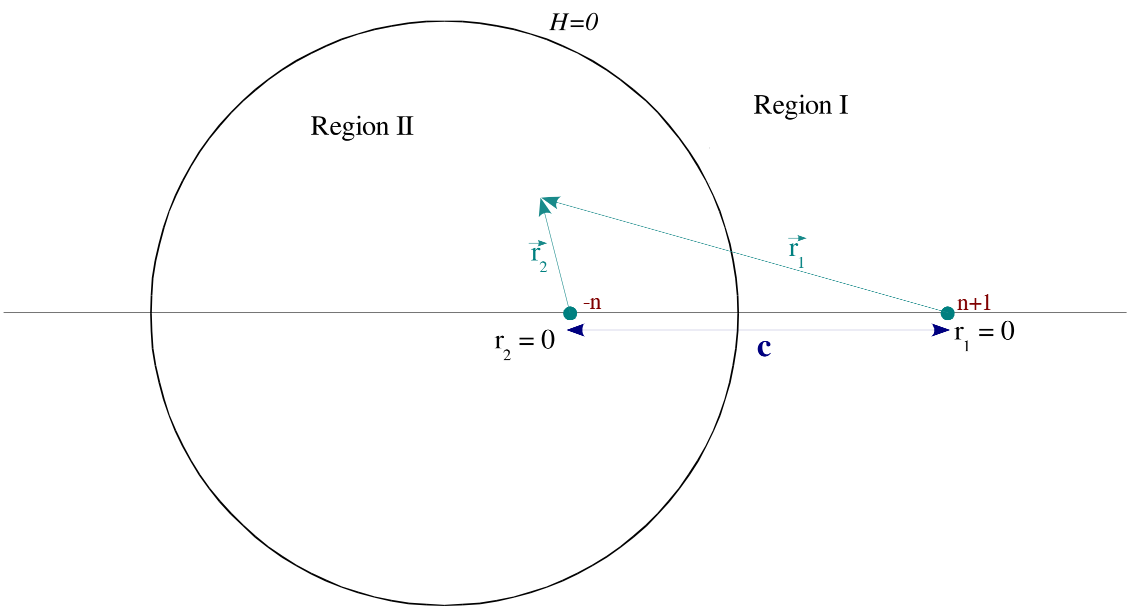

Figure 1: Basic 2-pole example has a spherical domain wall

Now that we have worked out a general formalism, let us illustrate it with the simplest example, the two pole solution; this reproduces the solutions of [11, 12, 13]. The simplest possible H that interests us has two poles and can be written in the following form (w.l.o.g we choose $n > 0$):

$$H = \frac{n+1}{r_1} + \frac{-n}{r_2} \tag{5.1}$$

where r_2 is defined as the distance in $\mathbf{R}^3$ from a point a distance $r_{12} = c$ below the origin on the ϕ symmetry axis; this gives HK a $U(1) \times U(1)$ isometry. Note that with this choice of the function H, Region II is a spherical region with its center just below the point $r_2 = 0$ (see figure). The function f is $f = -n\,r_1 + (n+1)\,r_2$ and has as its minimum value $-n\,c$.

For the dipoles we choose $d_i^{(1)} = -d_i^{(2)} = d_i$. The Z_i's and the net charges at infinity now take the simple form:

$$Z_i = 1 + \frac{Q_i}{4f}, \qquad Q_i = \frac{s^{ijk}}{2}\,\frac{d_j d_k}{n(n+1)}. \tag{5.2}$$

Notice that to guarantee positive Q_i's, we must impose that the d_i's all have the same sign.

From eq.(3.22), we see that the poles of K at $r_1 = 0$ and $r_2 = 0$ are:

$$\ell_1 = \frac{d_1 d_2 d_3}{16(n+1)^2}, \qquad \ell_2 = -\frac{d_1 d_2 d_3}{16n^2}. \tag{5.3}$$

This gives a simple form to k_0:

$$k_0 = \sum_i d_i \frac{r_2 - r_1}{4f} - \frac{\Pi_i d_i}{n^2(n+1)^2} \frac{(n+1)r_2 + nr_1}{16f^2}. \tag{5.4}$$

5.1 Solving the Constraints

In this simple case, there is a unique solution to the first constraints, eq.(4.25), which is the canonical solution:

$$c = \frac{1}{n^2(n+1)^2} \frac{\Pi_i d_i}{4\sum_i d_i} = \frac{1}{4n(n+1)} \frac{Q_1 Q_2 Q_3}{(Q_1 Q_2 + Q_2 Q_3 + Q_3 Q_1)}. \tag{5.5}$$

Using this value and the fact that the minimum value for f is $-n\,c$, the second constraint, eq.(4.26) can be easily checked. The final constraint, eq.(4.27), after some algebra, can be put in the form:

$$\Big[(M + 4f) + D/(2r_1 + 2r_2 + 2c)\Big]/4r_1 r_2 > 0 \tag{5.6}$$

where $M = Q_1 + Q_2 + Q_3$ and $D = Q_1 Q_2 + Q_2 Q_3 + Q_3 Q_1$. Each term in this product is manifestly positive, so we are guaranteed a smooth solution without CTC's or event horizons.

5.2 Features of the Solution

The angular momenta can be read off from eq. (3.30):

$$J_R = -d_1 d_2 d_3 \frac{2n+1}{n^2(n+1)^2} = \pm(2n+1)\sqrt{\frac{Q_1 Q_2 Q_3}{n(n+1)}} \tag{5.7}$$

$$J_L = 4c\sum_i d_i = \pm 4c\sqrt{\frac{n(n+1)}{Q_1 Q_2 Q_3}}(Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3) = \pm\sqrt{\frac{Q_1 Q_2 Q_3}{n(n+1)}}. \tag{5.8}$$

The whole of the solution is completely determined by an integer n and the three d_i's or, more physically: n, the three charges Q_i and a $\pm$ sign derived from a collective sign choice for d_i's. We can extend this solution to negative n's by taking $n \to -(n+1)$, which just flips the sign of J_R.

In general, if all three charges Q_i are of order Q, large relative to ℓ_P^2, then our system will be macroscopic for finite n, since $c \to Q/n(n+1) >> \ell_P^2$. If one of the charges is much smaller than the others, e.g. $Q_3 << Q_1, Q_2$, then c will be of order that smaller charge, and there will be high curvature regions if we don't also have $Q_3 >> \ell_P$.

To get a further understanding for the features of the two-pole solution, we draw the readers attention back to last part of eq.(5.2), which can be rewritten as

$$N_i = \frac{s^{ijk}}{2} \frac{m_j^{12} m_k^{12}}{n(n+1)} = \frac{s^{ijk}}{2} \frac{m_j m_k}{n(n+1)}. \tag{5.9}$$

Since the N_i are integers, the product of any two integer m_j's must be divisible by $n(n+1)$. Define $a = gcd\Big(m_3, n(n+1)\Big)$ and it's complement $b = n(n+1)/a$. The divisibility condition on the m_i's implies that there exist integers k_i and k such that

$$m_1 = b\, k_1,\ m_2 = b\, k_2,\ m_3 = a\, k_3, \ \text{ and }\ k_1\, k_2 = a\, k. \tag{5.10}$$

These yield the following relations

$$\begin{aligned} &N_1 = k_2\, k_3,\ N_2 = k_1\, k_3,\ N_3 = b\, k,\ N_1 N_2 = (k\, k_3)\, m_3, \\ &j_L = \frac{\pi}{4G_5} J_L = (k\, k_3), \qquad j_R = \frac{\pi}{4G_5} J_R = (2n+1)(k\, k_3). \end{aligned} \tag{5.11}$$

Clearly the quantity kk_3, let us label it N_s, plays a prominent role.

If $k_3 = 1$ and a divides n, there is a natural way of thinking about the numbers above. We take the third torus T_3 to have very small volume V_3. We then get a dual IIB description with N_1 and N_2 three-branes intersecting over a circle of radius ℓ_P^3/V_3 with N_3 units of momentum going around the circle. They can intersect over one long string, wrapped $N_1 N_2$ times around that circle or, more generally, $N_s = N_1 N_2/m_3$ substrings wrapped m_3 times. If we start with no momentum, we have:

$$j_L = j_R = N_s = N_1 N_2/m_3 = kk_3 \tag{5.12}$$

Following steps summarized in [12] we can apply a spectral flow shift of $2\tilde{n}$ to this CFT and get shifted values:

$$j_R \to j_R + 2\tilde{n}\, N_1 N_2, \qquad N_3 \to N_s\, \tilde{n}\, (\tilde{n}\, m_3 + 1) \tag{5.13}$$

Now define $n = \tilde{n}\, m_3$ and we get:

$$N_3 = n(n+1) N_s/m_3, \qquad j_L = N_s, \qquad j_R = (2n+1) N_s, \tag{5.14}$$

which fits nicely with eq. (5.11), except with the stronger assumption that m_3 divides n. Thus the states [12] are in a subclass of the generic two pole solution.

5.3 The Two Pole Solution as a Microstate

Since our original motivation was to find microstates for black holes and black rings, it is natural to ask if this geometry has the same asymptotics as one of those black objects.

Let us first consider the BMPV black hole [28]. Immediately, we run into a a problem: the basic solution above always has a non-zero j_L, so strictly speaking cannot be considered a microstate for the BMPV black hole in [28], which has $J_L = 0$. For large values of n and small values of N_s, however, it is possible to have J_R be macroscopically large in Planck units, while J_L remains of order the Planck length and can be coarse-grained away. In fact, the states that dominate the partition function for the CFT dual of the black hole (see [29] for a review) are in the single "long string" phase, which has $j_L = +1$. Taking this caveat into account, we see that in the large n limit $J_R^2 \to 4Q_1Q_2Q_3$, which means that our two -pole states are at best microstates of a zero area BMPV blackhole and as such cannot be considered as microstates for a proper BPS blackhole. We must look to other black objects.

By construction, the two-pole solution has $\vec{D}_i$'s which are parallel. This feature potentially qualifies it as a black ring microstate. Black rings, however, have more parameters than black holes, so we have to be careful in matching these. If we use the nomenclature in [5], we see that the parameters q_i which appear in the expression for the horizon area only appear as $R^2 q_i$ in the asymptotics, where R is the inner radius of the ring. This leads to an ambiguity, since we match

$$4cd_i = R^2 q_i. \tag{5.15}$$

As it turns out, q_i and d_i are quantized in the same units, so we have

$$R^2/4c = d_i/q_i = a/b, \qquad a, b \in \mathbf{Z}. \tag{5.16}$$

The black ring solutions require

$$\left(Q_i = \frac{s^{ijk}}{2} \frac{d_j d_k}{n(n+1)} \right) \geq \frac{s^{ijk}}{2} q_j q_k \quad \Rightarrow \quad \frac{a^2}{b^2} \geq n(n+1). \tag{5.17}$$

Without going into too much detail, if we compare our expressions for the angular momenta eq.(5.7) and eq.(5.8) with those in [5, 16], we find that the expressions for J_L match (this match is independent of N) if we use eq.(5.15) but that our J_R is always too small to qualify for a black ring. Only in the limit where we saturate the bound in eq.(5.17) and n is very large do we start converging on the same value for J_R. At that point, our solution ends up sharing the asymptotics of a zero-area ring. This is a very similar story to that of the BMPV black hole.

In conclusion, the two-pole solution can at best appear as a microstate for "black" rings and "black" holes with zero horizon areas. In the context of the CFT microstates connection we developed earlier, this is none too surprising, as those states are unique given their charges and angular momenta. We also see that our supergravity solution does not appear to have any adjustable parameters, hence should not belong to the very numerous, i.e. entropic, class of states that should match a black object.

5.4 The Domain Wall

The surface $H = 0$ has a radius:

$$R_{DW}^2 = 4r_{DW} = 4c\,\frac{Q_1 Q_2 Q_3}{J_L J_R}. \tag{5.18}$$

In the region near $r_2 = 0$ with radius of order $R = \sqrt{c}$, the functions Z_i are locally constant and $k \propto r_2\,\sigma$, which means that the local metric is a Z_n orbifold of a Gödel universe. This suggests that our solution is in fact the un-smeared version of the hypertube speculated about in [30] and is in fact a smooth resolution of the type of domain wall illustrated there.

Given that the hypertube-like solution we have found has completely smooth supergravity fields except for mild orbifold singularities, one is tempted to ask what physical features the surface at $H = 0$ exhibits, if at all, to mark it's presence. An associated issue is the question of just where the membrane charge detectable at infinity is sourced.

To answer the first question, it is interesting to consider a probe membrane in our background wrapping the z_1, z_2 and t directions. If we include a small velocity v^0 along the σ fiber and $\vec{v}$ in the base space, we see that the probe action in the static gauge has an expansion which looks like:

$$\begin{aligned} &e_1 \int \Big[Z_1^{-1}\sqrt{(1 + k_0 v^0 + k_a v^a)^2 - Z_1 Z_2 Z_3 (H^{-1} v_0^2 + H g_{ab} v^a v^b)} \\ &-Z_1^{-1}(1 + k_0 v^0 + k_a v^a) + 2a_{i0} v^0 + 2a_{ia} v^a \Big] d\lambda. \end{aligned} \tag{5.19}$$

Clearly, there is no potential term, as can be expected for a BPS configuration. Away from the $H = 0$ domain wall, we expand the action in small powers of the velocity, and we get:

$$e_1 \int \Big[-\frac{1}{2}\, Z_2 Z_3 \left(H^{-1} v_0^2 + H g_{ab} v^a v^b\right) - 2(a_{i0} v^0 + a_{ia} v^a) + \mathcal{O}(v^3) \Big] d\lambda. \tag{5.20}$$

This is the action for a charged particle on HK traveling in a magnetic field, albeit with variable mass. Near $H = 0$, the mass parameter and magnetic field blow up! At this point, we can no longer work in the small v approximation and must go back to the full action, itself always finite.

Another way to analyze the action as $H \to 0$, is to realize that ∂_t becomes lightlike turning our static gauge into a light cone gauge. Expanding the worldline tangent for our wrapped membrane "particle" about the lightlike velocity vector ∂_t gives us a theory with exact Galilean invariance. The momentum p_t becomes the lightcone energy, and since the only mixed term in the line element looks like $dt\,\sigma$, τ seems to be the natural lightcone time variable. The interesting thing here is that τ is periodic, and ∂_τ is spacelike. Thus our

membrane worldtheory takes on aspects of a finite temperature non-relativistic system! It is interesting to speculate about what this might mean for the process of "heating up" our solution to make it a microstate of a finite temperature near-BPS black hole, something like the states considered in [15]. We leave this for future work.

In summary, the domain wall at $H = 0$ leads to some interesting behavior for the world-volume on our probe brane, yet there is still no real discontinuity there and no vanishing of the probe kinetic term as for an enhancon [31], in fact just the opposite.

The charge in the hypertube has dissolved away, so where should we think of it existing? In our case, an alternate scenario to charges localized on a domain wall appears. We see instead a situation similar to that of the geometric transition in [32, 33], where a large number of D-branes wrapped on an S^3 at the tip of a cone over S^2 is replaced by flux on a non-contractible S^2 at the tip of a cone over S^3. We can take a decoupling limit for our solution by removing the 1's in the Z_i's to get a five-dimensional space which is asymptotically an orbifold of $AdS_2 \times S^3$. The infrared limit of the holographically dual theory should reflect the appearance of this geometric transition.

6 Adding More Poles

Clearly, if we want to achieve a more general solution which could have the same asymptotics as BPS black hole or black ring with a non-zero horizon area, the next step is to consider a solution with more poles, starting with three poles. We will see that this type of solution has more adjustable parameters, making it a more likely candidate.

6.1 Simple Three Pole Solutions

With three poles to play with, the simplest scenario is for H to have two poles with residue $n_1 = n_2 = +1$ and one with residue $n_3 = -1$. With these choices, the solution is completely smooth! We define the three radial distances from these poles as r_1, r_2 and r_3, and corresponding dipoles $d_i^{(1)}, d_i^{(2)}$ and $-(d_i^{(1)} + d_i^{(2)})$. One nice feature is that we can now vary the dipoles and J_L by our choice of pole arrangements

We can further simplify our solution by working with dipoles that are diagonal in the three $U(1)$'s. This will guarantee that the dipole vector's $\vec{D}_i$ will all be parallel, and thus allow comparison with BPS black rings, especially the ones in [4].

We start with probably the simplest example. $d_i^{(1)} = d_i^{(2)} = d$ and $d_i^{(3)} = -2d$. Then we

find the set of equations eq. (4.25) become simply

$$r_{13} = r_{23} = \frac{d^2}{12}, \tag{6.1}$$

with no restrictions on r_{12}. We must now satisfy $Z_i H > 0$, eq. (4.26). This condition becomes

$$4(r_2 r_3 + r_1 r_3 - r_1 r_2) + d^2(r_1 + r_2) > 0. \tag{6.2}$$

Using our values for r_{13} and r_{23} one can easily show this equation is satisfied at the three poles. It is also trivially satisfied at asymptotic infinity. Though it is difficult to show analytically, a numerical analysis confirms that it is satisfied everywhere for any value of r_{12}. Let us examine the implications of this. Without loss of generality we choose to place pole 3 at the origin, and pole 1 on the z-axis. Then, since there are no restrictions on r_{12} we are free to place it anywhere on a circle of radius $r_{23} = r_{13} = d^2/12$. Let the angle 132 between segments 13 and 23 be called ψ. Then we find

$$\vec{D}_i = \frac{d^3}{12}[\hat{z}(1 + \cos\psi) + \hat{x}\sin\psi]. \tag{6.3}$$

One can read off the asymptotic charges for this solution. They are

$$Q_1 = Q_2 = Q_3 = 2d^2, \quad J_R^2 = 36d^6, \quad J_L^2 = 2d^6(1 + \cos\psi). \tag{6.4}$$

One can easily see that while Q_i and J_R are independent of our pole arrangement, J_L is very sensitive to it. We find that $0 \leq J_L^2 \leq 4d^6$, where the minimum value is reached when the negative pole (3) is in the middle of the two positive poles. The maximum value is reached when the two positive poles lie on top of each other, returning us to the 2-pole example from the previous section with $n = 1$. Note also that even in this very simple three pole case, the last condition, eq. (4.27) is quite challenging; and we will not check it here.

For the purposes of finding microstates for black objects, adding a third pole already seems to help. For example, we can explicitly set J_L to zero, which is characteristic of BMPV microstates. Unfortunately J_R is still too large and saturates the BMPV bounds just as in the two-pole case. On the black ring side, things look more promising. As we adjust the angle ψ the three dipoles have variable magnitude $2d\cos\psi/2 \leq 2$, matching to a class of black rings with variable J_R, a function of the dipoles and smaller as we increase ψ. The upshot is that for $0 < \psi < \pi$, we can now adjust the constant a/b in eq.(5.16) so that the three-pole solution's quantum number j_R matches that of the black ring with the same dipoles and charges at infinity. Hence, modulo the CTC condition in eq.(4.27), we have identified supergravity microstates for black rings with finite-sized horizons.

6.2 Comments on the General Case

As we have seen, the three pole case allows us more freedom in positioning our poles. This freedom tends to only vary J_L and the $\vec{D}_i$'s, hence it does not generate a large class of states with the same asymptotics. Ideally, we would like to be able to fix the Q_i's, $\vec{D}_i$'s, J_L and J_R and still find many solutions. It seems reasonable to believe that adding more poles should allow us to have much larger moduli spaces of solutions, including substantial subspaces with the same Q_i's,$D_i's$ and $J_{L,R}$; further work towards understanding BPS black ring and black hole microstates will requiring developing a better understanding of the general n-pole solution.

7 Discussion

The general picture that is emerging from our solution is that of generic five dimensional BPS three-charge black hole and black ring microstates coming from a harmonic function with a large number of poles. These solution are characterized by discrete choices, the d_i^{pq}'s, as well as continuous ones, the relative positions of the points $\vec{x_p}$. This dichotomy is reflected in the angular momenta, with J_R clearly discrete in all cases while J_L varies continuously requiring explicit quantization. Similarly, we see one set of exact constraints eq.(4.25) complemented by inequalities, inexact constraints, from eqs.(4.26-4.27).

It is likely that the dichotomy above arises from the explicit $U(1)$ invariance that permeates our solution, arising from our ansatz for the pseudo-hyperkahler base space in the Gibbons-Hawking form. This raises the question of what more general pseudo-hyperkahler base spaces look like. These general spaces could also have integer quantized fluxes on two-cycles, but these need no longer share the same $U(1)$ symmetry and so the asymptotic angular momenta would both vary continuously.

The exact $U(1)$ symmetry which we have, non-generic in five dimensions, can become an asset if we choose to use it to reduce to four dimensions. This can be done by simply adding a term of the form $1/R^2$ to the harmonic function, and relaxing the condition on the sum of the residues; most of our analysis will survive unchanged. This yields a substantial generalization of the four dimensional solutions in [21, 6, 22]. In those papers, solutions appear with a KK-monopole charge which is basically $\sum_p n_p$, but only allowing positive residues! Modulo constraints of the form in eqs.(4.25-4.27), the possibility of adding negative residues to the poles of H greatly broadens this class of supersymmetric four dimensional solutions. If we make τ the M-direction, the appearance of negative poles corresponds to having anti-D6-branes while keeping the supersymmetries of D6-branes, this is similar to

what happens in [34]. The appearance of both D-branes and anti-D-branes in a black hole microstate is something we have learned to expect in near-extremal systems to but is new to extremal ones.

We close by stressing what we feel is the most important idea emerging from our analysis: supergravity solutions dual to wrapped brane bound states replace a core region which naively would have a naked singularity or a horizon with a core region containing a "foam" of new topologically nontrivial cycles. This is a fascinating combination of Vafa etal.'s geometric transitions picture [32, 33] and melting crystal space-time foam picture [35, 36] which will certainly attract future attention.

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